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Program / Major: Mathematics

Mathematical Modeling and Simulations of Mobile Robots

Abstract

- Simulations of hexapod (six-legged) and quadruped (four-legged) robots have important applications for mobile robot design for process automation, industrial applications, and deriving algorithms for walking styles.
- Legged mobile robots can traverse uneven terrain and use artificial intelligence to plan their safe foothold positions to navigate their environment.
- Their outstanding mobility makes the mobile robotic platforms perfect for space exploration and automated search and rescue deployment.
- Here, we have used the Webots robotic simulations to study six-legged hexapod mantises and four-legged robots that mimic dog-like movements.
- We have analyzed the simulated gaits and poses using rigid-body inverse kinematics and symmetry analysis.
- The hexapod robot moves using an alternating tripod-like gait where three of its legs move at a time while the other three remain stationary.
- The hexapod robot has great dynamic stability for uneven terrain and can move more legs than a quadruped robot.
- This research project serves as a sophisticated platform for the hands-on application of mathematical simulation methods in real-world solutions.

Objectives / Points

- Robotic Simulations of six and four and six legged mobile robots have important applications for mobile robot design for process automation and industrial applications.
- Legged Mobile robots can traverse uneven terrain and use artificial intelligence to plan their safe foothold positions to navigate their environment.
- These outstanding mobility capabilities make robotic platforms perfect for deployment for space exploration, automated search and rescue and inspection.
- We have analyzed the observed Robot motion for the hexapod and quadruped robot obtained in the simulations using mathematical modeling, symmetry analysis and inverse kinematics
- Through trigonometric studies in combination with linear algebra we were able to analyze the motion of the hexapod robot.
- We were able to analyze the motion of the hexapod robot by treating it as a rigid unit without constraints which had 6 degrees of freedom in 3 dimensions: three translational and three rotational degrees of freedom.
- To study the gait and pose of the robots, we used the Webots code to simulate the hexapod and 4-legged robot and their movements.
- Webots – Open Source Program
  https://cyberbotics.com/

Webots
robot simulation

Physics Background

describe the relationship between forces/torques and motion (in joint space or workspace variables)

two possible goals:

1. Given motion variables (e.g. $\vec{\theta}, \dot{\vec{\theta}}, \ddot{\vec{\theta}}$ or $\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}$), what joint torques ($\vec{\tau}$) or end-effector forces ($\vec{f}$) would have been the cause? *(this is inverse dynamics)*

2. Given joint torques ($\vec{\tau}$) or end-effector forces ($\vec{f}$), what motions (e.g. $\vec{\theta}, \dot{\vec{\theta}}, \ddot{\vec{\theta}}$ or $\vec{x}, \dot{\vec{x}}, \ddot{\vec{x}}$) would result? *(this is forward dynamics)*

- Given positions and angles, calculate force and torque

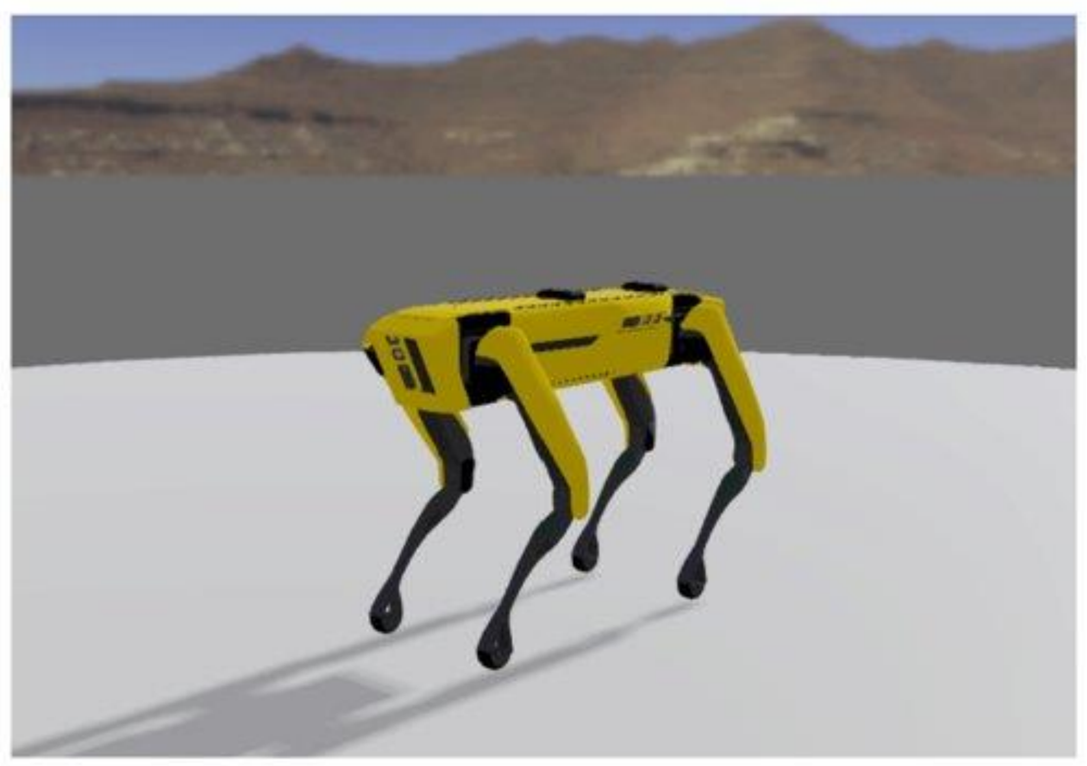
Force = **F** = $m\frac{d^2\boldsymbol{x}}{dt^2}$

Torque τ = r x F

vector cross product

Hexapod Robot

Quadruped Robot



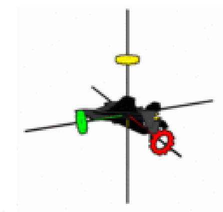
Spot: Boston Dynamics robot in Webots

Mathematical Example

Translations

The translation about x, y, and z axes of l is then in the form:

$$T_x = \begin{bmatrix} l \\ 0 \\ 0 \end{bmatrix}, \quad T_y = \begin{bmatrix} 0 \\ l \\ 0 \end{bmatrix}, \quad T_z = \begin{bmatrix} 0 \\ 0 \\ l \end{bmatrix}$$



Rotation matrices

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Rotation by angle θ about the z-axis

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$ Rotation by angle θ about the y-axis

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$ Rotation by angle θ about the x-axis

Transformed coordinates (x',y',z') for rotation by angle θ about the z-axis and translation of units along x

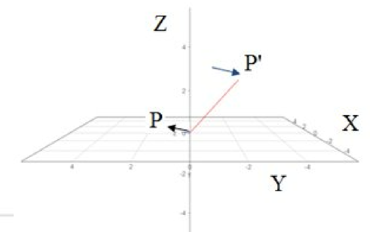
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_z(\theta)\begin{pmatrix} x \\ y \\ z \end{pmatrix} + T_x$$

Coordinate Transformation

Consider the point P(1,-1,0) rotated by 30° about the x-axis and translation by (2,-1,3)

Rotation matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

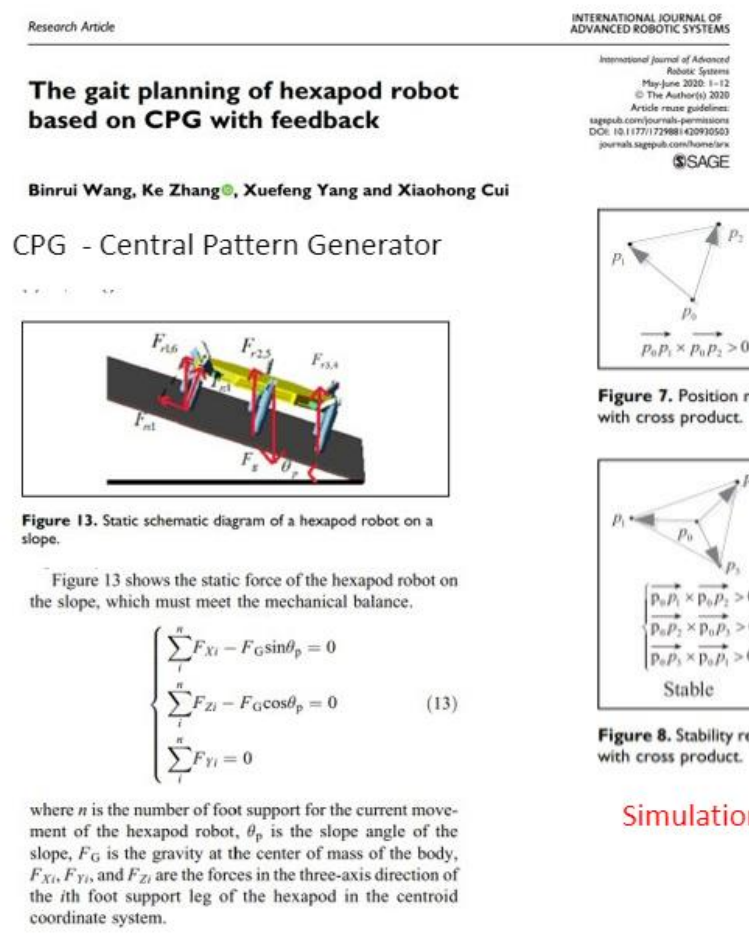


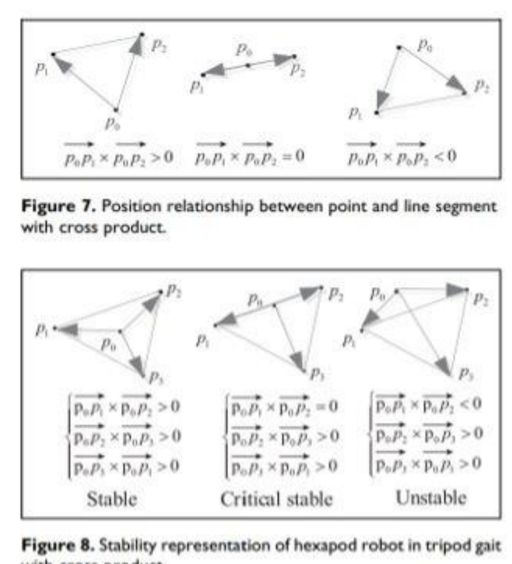
Transformation: (Rotation only)

$$(x, y, z) \longrightarrow \left(x, \frac{\sqrt{3}\,y}{2} - \frac{z}{2}, \frac{y}{2} + \frac{\sqrt{3}\,z}{2}\right)$$

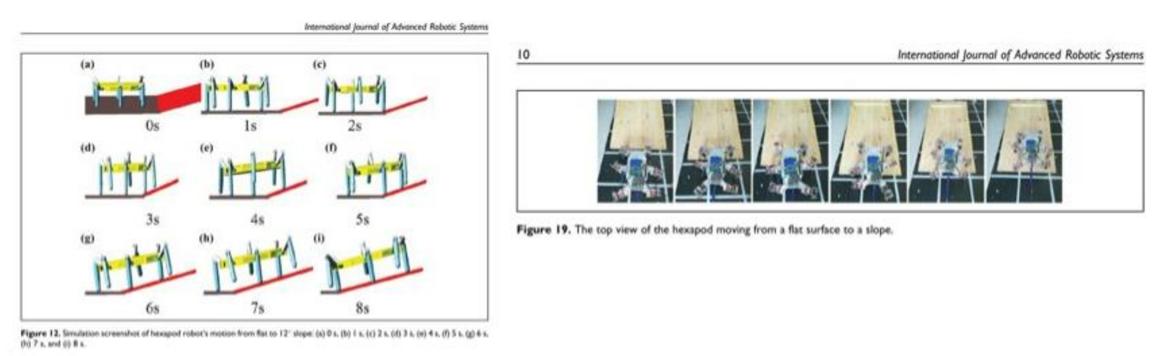
Transformed point P' (Rotation +Translation): $\left(3, -1 - \frac{\sqrt{3}}{2}, \frac{5}{2}\right)$

Hexapod Gait Research





Simulations using Matlab



- Simulations by Wang *et al. (2020)*, shows that tripod gait can be used to climb uneven terrain
- Wang *et al. (2020)*, used cross-product criteria with simulations to study stability and found that the tripod gait gives the hexapod great stability and helps also with fast locomotion on uneven terrain.

Conclusion

- We studied the gait and pose of bioinspired hexapods and 4-legged robots obtained using the Webots simulation code.
- Both 4 and 6- legged robots can navigate uneven terrain. The hexapod robot has greater dynamic stability for uneven terrain and can move a greater number of legs as compared to the quadruped.
- This research project serves as an elegant platform for hands on application of mathematical and simulation methods for studying the gait and stability of legged mobile robots.
- These studies can be applied for the safety design of computer-controlled walking robots for radioactive waste management, space exploration, nuclear power stations, and other industrial applications.

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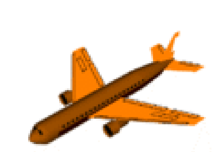

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